Ising model in an oscillating magnetic field: Mean-field theory

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I consider the dynamics of a soft-spin Ising magnet, subject to the time-varying external magnetic field $h(t)=h\cos\Omega t$. The system is modeled with a time-dependent Ginzburg-Landau equation and is studied at a mean-field level. The time-averaged magnetization ($\mathcal{M}$) acts as the order parameter and divides the temperature-h plane into two phases. In contrast to a previous result that predicted a dynamical tricritical point separating a line of continuous and discontinuous transitions in $\mathcal{M}$, I find that the transition is always continuous. The previous work utilized Glauber dynamics which, it is here argued, becomes inapplicable near the phase boundary line due to a critical slowing down.

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I. INTRODUCTION

There is in general little that is known about systems far from equilibrium. In order to gather such an understanding I will focus on a particular system forced from equilibrium by the action of a time-dependent field. Specifically, the model is a set of Ising spins subject to the external magnetic field $h(t)=h\cos\Omega t$; the magnetization is taken to satisfy a time-dependent Ginzburg-Landau equation. In the absence of the external field the system is described by the usual ϕ^4 Hamiltonian used to model second-order phase transitions. However, when $h(t)$ is present it is no longer valid to use an equilibrium (Boltzmann) distribution in calculating fluctuations, which is why the modeling is done at the level of the equation of motion. In the past [1], this system has been modeled with Glauber dynamics [2], which utilizes the equilibrium distribution. In the case when the period of oscillation of the field is much greater than the relaxation time of the magnetization this is a reasonable approximation. However, this is only the case for a limited range of parameters, which does not include continuous phase-transition lines; there, a critical slowing down invalidates the use of Glauber dynamics.

Aside from its utility in approaching a general understanding of nonequilibrium systems, this model also describes certain types of measurements on spin systems [5]. For example, upon subjecting the sample to an oscillating magnetic field and measuring the induced magnetization, one can calculate the (frequency-dependent) response and correlation functions. As will be discussed, what is sought is something that is useful for large external fields, something beyond a lowest-order response. Also, this model is relevant for more practical situations, such as for machines which thermally anneal metallic samples by subjecting them to an oscillating magnetic field [6].

The outline of the paper is as follows. In Sec. II the equation of motion for the magnetization is given, and assumptions used in applying it to a physical system are discussed. In Sec. III, after mentioning the order parameter that has been used for this system, the stability of the magnetization in each phase is discussed; this is used to determine the phase boundary. In Sec. IV the order parameter is found for a range of parameters and plotted in the temperature-h plane. There it is shown that the time-averaged magnetization changes very suddenly, but in fact not discontinuously. The behavior of the phase boundary line as a function of the parameters is given. In Sec. V two simple analytical approaches are used to find the magnetization in each of the two phases and are compared with the numerical solution. In Sec. VI a comparison is made to a previous work which (implicitly) predicts a discontinuous phase transition. It is argued that their work is in error, as it relies on Glauber dynamics, which is explicitly shown to break down near the phase boundary line (due to critical slowing down). Finally, a summary is presented in Sec. VII.

II. MODEL

The modeling of the system is based on that used when there is no external magnetic field. Beginning with the (coarse-grained) time-dependent Ginzburg-Landau description, the magnetization is linearly coupled with an oscillating magnetic field. As is always (at least implicitly) done when modeling with stochastic equations, it is assumed there is a clear separation of time scales between that of the noise and the characteristic scales of the magnetization and the oscillating field. For an Ising magnetization the equation of motion is

$$\partial_t\psi=-\Gamma_0\frac{\delta H}{\delta\psi}+\nu\ , \tag{2.1}$$

$$H[\psi]=\int d^dr\{\tfrac{1}{2}[r_0\psi^2+(\nabla\psi)^2]+(u_0/4)\psi^4-h(t)\psi\}\ . \tag{2.2}$$

The noise is zero-mean Gaussian with covariance

$$\langle\nu(r,t)\nu(r',t')\rangle=2\Gamma_0\delta(r-r')\delta(t-t')\ , \tag{2.3}$$

where $\langle\ \rangle$ denotes a noise average. It is not known how to rigorously justify this choice of the noise correlation;

further specification would require knowledge of the (as of yet unknown) distribution function [7]. $H[\psi]$ is the usual ψ^4 Hamiltonian for the order-parameter field $\psi(r,t)$, the local magnetization. r_0 is the reduced temperature and is $\propto T-T_{c0}$; T and T_{c0} are the temperature and (unrenormalized) critical temperature, respectively. Also, the quartic coupling u_0 is a positive constant and the period of oscillation equals $\tau=2\pi/\Omega$. The (unrenormalized) kinetic coefficient of the magnetization is Γ_0, which can be found through linewidth measurements of the ferromagnetic resonance. Typical (noncritical) spin relaxation times are on the order of 10^{-8} sec [8].

As the external field oscillates, it drives the magnetization in the sample to oscillate as well. In a real physical system this leads to dissipation; an amount of heat $\sim\int_0^\tau\psi(dh/dt)dt$ is deposited in the sample with each period. Because of this, the above model is only physically applicable to systems that are sufficiently thin along one dimension (so that heat may diffuse away to an adjacent thermal bath). This situation is not unique to this treatment; it also arises with linear-response theory [9]. The lesson is that another field is needed to account for the heat and so provide a thermodynamically complete description.

In this paper the noise- (and time-) averaged magnetization is sought. After dependences on initial conditions have been forgotten (assuming a noncritical system), it is expected that the magnetization will settle into a periodic, spatially independent function $M(t)$. The equation of motion that $M(t)$ satisfies can be found perturbatively by shifting $\psi\rightarrow\psi+M(t)$ and then demanding $\langle\psi\rangle=0$ [10]. The lowest-order contribution to this condition is simply

$$\partial_t M=-\Gamma_0[r_0M+u_0M^3-h(t)]\ ; \tag{2.4}$$

this also follows from Eq. (2.1) by ignoring the noise and gradient terms. This equation will be used to study the system at a mean-field level.

III. PHASE BOUNDARY

The periodic magnetization $M(t)$ (with period $\tau=2\pi/\Omega$) may be averaged over one period to give the order parameter

$$\mathcal{M}=\frac{1}{\tau}\int_t^{t+\tau}M(t')dt'\ , \tag{3.1}$$

which becomes independent of the time t once the initial conditions have been forgotten [11]. As has been discussed [1], there are two phases that appear; one is characterized by $\mathcal{M}=0$, the zero (Z) phase, and the other by $\mathcal{M}\neq0$, the nonzero (NZ) phase [12]. The stability of the magnetization will now be studied in order to find the phase boundary separating the Z and NZ phases.

Denoting a time-average with an overbar, $M(t)$ is separated into its constant ($m_0=\overline{M}$) and purely oscillating ($\tilde{m}=M-m_0$) parts. Making the substitution $M=m_0+\tilde{m}$ into Eq. (2.4) and averaging over one period, it follows

$$u_0\overline{\tilde{m}^3}+3u_0m_0\overline{\tilde{m}^2}+r_0m_0+u_0m_0^3=0\ . \tag{3.2}$$

Starting in the NZ phase and then approaching the phase boundary (i.e., $m_0\rightarrow0^+$) gives

$$r_0+3u_0\overline{\tilde{m}^2}=0\ , \tag{3.3}$$

since $\overline{\tilde{m}^3}=0$ in the Z phase. Here it was implicitly assumed that it is possible to take the limit of m_0 to zero, i.e, that m_0 does not change discontinuously. However, as will be seen later, numerical simulation shows that m_0 always does change continuously, and so Eq. (3.3) always characterizes the Z-NZ phase boundary.

The equation of motion $\tilde{m}$ satisfies is found by substituting $M=m_0+\tilde{m}$ into Eq. (2.4) and then using the constraint of Eq. (3.2) to get

$$\partial_t\tilde{m}=-\Gamma_0\{(r_0+3u_0m_0^2)\tilde{m}+3u_0m_0(\tilde{m}^2-\overline{\tilde{m}^2})+u_0(\tilde{m}^3-\overline{\tilde{m}^3})-h(t)\}\ . \tag{3.4}$$

This can be solved (perturbatively, for example) to get an equation for $\tilde{m}$ as a function of m_0. In turn, that may be used in conjunction with Eq. (3.2) to arrive at a closed equation, involving only m_0. This method of calculating the average magnetization m_0 will be pursued in Sec. V.

The (linear) stability of M with respect to a perturbation $\epsilon(t)$ is now considered. After substituting $M=m_0+\tilde{m}(t)+\epsilon(t)$ into Eq. (2.4), and using Eq. (3.4), it follows

$$\partial_t\epsilon(t)=-\Gamma_0\epsilon(t)\{r_0+3u_0(m_0+\tilde{m})^2\}+O(\epsilon^2)\ . \tag{3.5}$$

The time-independent terms in the curly brackets are used to obtain the relaxation rate ω to an infinitesimal perturbation:

$$\omega=\Gamma_0[r_0+3u_0(m_0^2+\overline{\tilde{m}^2})]\ ; \tag{3.6}$$

in the Z and NZ phases, this becomes, respectively,

$$\omega_Z=\Gamma_0[r_0+3u_0\overline{\tilde{m}^2}]\ , \tag{3.7}$$

$$\omega_{NZ}=\Gamma_0[r_0+3u_0(m_0^2+\overline{\tilde{m}^2})]\ . \tag{3.8}$$

It is noted that the vanishing of the relaxation rates corresponds to the phase boundary, as defined by Eq. (3.3).

Although spatial coordinates do not appear explicitly in Eq. (2.4), $M(t)$ is derived from Eq. (2.1) and so carries the same dimension as $\psi(r,t)$; thus it is sensible to make a dimensional analysis in terms of length scales. Aside from the lengths associated with static quantities [i.e., $L_\xi=(|r_0|)^{-1/2}$ and $L_h=(u_0h^2)^{-1/2}$], there is a new length scale $L_\Omega=(\Gamma_0/\Omega)^{1/2}$. Normally Γ_0 can only enter as a length through the combination Γ_0t. However, since there is now the characteristic time dependence Ω^{-1}, it is possible to have Γ_0 (or L_Ω) appear in time-independent quantities. L_Ω may be interpreted as the correlation length, after quenching and waiting a time $1/\Omega$ (assuming exponential growth, which is only reasonable initially). The appearance of such a length in static quantities does not occur in simpler near-equilibrium models, where

the statics can be shown to be independent of the dynamics [13]. This should be compared with the work of Onuki and Kawasaki [14] on fluid under a shear flow. There it was seen that the shear induced a length scale which represented the largest size a fluctuation could grow to before being destroyed by the shear. A similar situation also arises with liquid crystals in a shear flow [15].

It is possible to obtain a scaling behavior of the Z-NZ boundary curve by considering two limiting cases in comparing L_Ω to L_ξ and L_h. First, upon substituting $M(t)=h\tilde{m}(t)$, Eq. (3.3) may be rewritten as

$$-r_{0c}=3u_0\overline{M^2}\left[r_{0c},\frac{\Omega}{\Gamma_0},u_0,h\right]$$
$$=3u_0h^2\overline{\tilde{m}^2}\left[r_{0c},\frac{\Omega}{\Gamma_0},u_0h^2\right]. \tag{3.9}$$

In the limit $L_\Omega \ll L_\xi$ and L_h, both r_{0c} and u_0h^2 are small. Also, from Eq. (3.4) it seems reasonable that $\overline{\tilde{m}^2}$ does not vanish as r_{0c} and h approach zero. Thus, scaling out the dimension of $\overline{\tilde{m}^2}$ with L_Ω, it follows, for small r_{0c}, that

$$-r_{0c}\sim u_0h^2\left[\frac{\Gamma_0}{\Omega}\right]^2. \tag{3.10}$$

In the limit $L_h,L_\xi \ll L_\Omega$ (taking without loss of generality $L_h \ll L_\xi$), r_{0c} may be written as (now scaling with respect to L_h)

$$-r_{0c}=u_0h^2(u_0h^2)^{-2/3}\times f\left[\frac{r_{0c}}{(u_0h^2)^{1/3}},\frac{\Omega}{\Gamma_0}\frac{1}{(u_0h^2)^{1/3}},1\right], \tag{3.11}$$

where f is a dimensionless scaling function; from this it is easily seen that

$$-r_{0c}\sim(u_0h^2)^{1/3} \tag{3.12}$$

for large $-r_{0c}$. It must be remembered that both of these scaling results cannot be proved from these arguments, since it must be known that $\overline{\tilde{m}^2}$ is well behaved in these limits, as has been assumed. However, they will be vindicated in the next section, where these behaviors are explicitly computed (see Fig. 2).

It is easy to show that all phase boundary curves in the r_0-h plane can be scaled onto a single curve. Scaling $\overline{M^2}$ as

$$\overline{M^2}=\frac{-r_0}{u_0}g\left[r_0\frac{\Gamma_0}{\Omega},h\left[u_0\left[\frac{\Gamma_0}{\Omega}\right]^3\right]^{1/2}\right], \tag{3.13}$$

with g a dimensionless function, the equation for the phase boundary [i.e., Eq. (3.3)] becomes $3g=1$. This implies that if we work in the scaled coordinates $\tilde{r}_0=r_0(\Gamma_0/\Omega)$ and $\tilde{h}=h[u_0(\Gamma_0/\Omega)^3]^{1/2}$ there will be a single phase boundary curve for all choices of Γ_0 and Ω. (Note that when noise is included, $\overline{M^2}$ must be expanded in terms of u_0 and u_0h^2, as opposed to just u_0h^2.)

IV. NUMERICAL SOLUTION

A fourth-order Runge-Kutta scheme is used to solve for $M(t)$, with $dt=0.01$, and 400 points per period of oscillation. Near the phase boundary, the time step was in some cases decreased to $dt=0.0001$; different ratios of Ω/Γ_0 were found by varying Γ_0. Also, the time step was increased and decreased by a factor of 2 to check the reliability of the simulation. After letting the system relax from its initial condition for a sufficient time (which varied greatly depending on the nearness to the Z-NZ boundary), the magnetization was measured for several periods. That magnetization is then used as the initial condition for the next run at a higher h. No hysteresis was seen as h was increased and decreased through the boundary.

The Z-NZ phase boundary is shown in Fig. 1 for three different values of Ω/Γ_0. These curves were scaled onto one universal curve (see Fig. 2) using the scaled coordinates $\tilde{r}_0,\tilde{h}$, which were argued for earlier. For low temperatures, the general behavior of $\mathcal{M}$ for small and large h is understandable. When h is small the magnetization can be thought of as being determined by the two equilibrium wells, with minima at $\pm(-r_0/u_0)^{1/2}$ (disregarding fluctuation effects). Then the external field merely adds a small oscillating component to the constant part. On the other hand, when the external field is large, one of the minima may disappear (for part of the period), allowing the magnetization to oscillate back and forth equally between the positive and negative wells. The scaling behavior of $r_{0c}(h)$ that was alluded to earlier can be checked by making a log-log plot of the data of Fig. 2; the result is

$$-r_{0c}\approx\begin{cases}5.08u_0h^2(\Gamma_0/\Omega)^2, & r_0\to 0\\ 1.89(u_0h^2)^{1/3}, & r_0\to-\infty\end{cases}, \tag{4.1}$$

(4.2)

which is in agreement with the scaling behaviors of Eqs. (3.10) and (3.12).

A three-dimensional plot of $\mathcal{M}$ is given in Fig. 3, offering an overview of the phase diagram. Of note is the

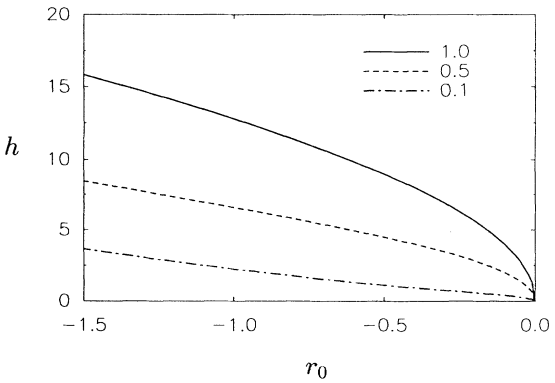

FIG. 1. Plots of the phase boundary lines separating the $\mathcal{M}=0$ (Z phase) and $\mathcal{M}\neq 0$ (NZ phase) phases for the three cases of $(1/2\pi)(\Omega/\Gamma_0)=0.1$, 0.5, and 1.0.

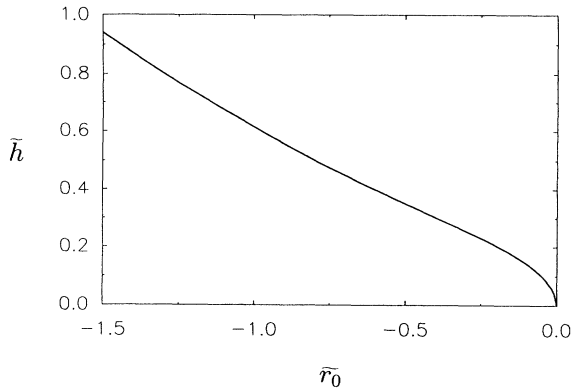

FIG. 2. The three curves of Fig. 1 appear as one if they are plotted on the scaled axes $(\tilde{r}_0,\tilde{h})$, where $\tilde{r}_0=r_0(\Gamma_0/\Omega)$, $\tilde{h}=h[u_0(\Gamma_0/\Omega)^3]^{1/2}$ [see Eq. (3.13)].

sharp dropoff that develops in $\mathcal{M}(h)$ at low temperatures. However, it is easier to ascertain changes in the slope of $\mathcal{M}$ by taking slices of it, say, at several different temperatures (see Fig. 4). Although the slope in $\mathcal{M}$ vs h becomes very steep there, a close inspection never revealed a discontinuous jump. A steep slope could also be found at a fixed r_0 upon lowering Ω/Γ_0 (as required by the dependence of $\tilde{r}_0$ and $\tilde{h}$ on Ω/Γ_0), as in Fig. 5(a). An expanded view of $\mathcal{M}(h)$ near the phase boundary for the most sharply increasing $\mathcal{M}$ is shown in Fig. 5(b). This initial slow increase in $\mathcal{M}$ occurs over a smaller range in h for lower temperatures, but was never found to vanish. The similarities in the general shape of this curve and Fig. 4 can be understood from the scaling relation (which follows easily from dimensional analysis)

$$\mathcal{M}\left[sr_0,\frac{\Gamma_0}{\Omega},h\right]=s^{1/2}\mathcal{M}\left[r_0,s\frac{\Gamma_0}{\Omega},\frac{h}{s^{3/2}}\right], \tag{4.3}$$

where use was made of the fact that u_0 always enters with h as u_0h^2. Thus, aside from a scaling of the $\mathcal{M}$ and

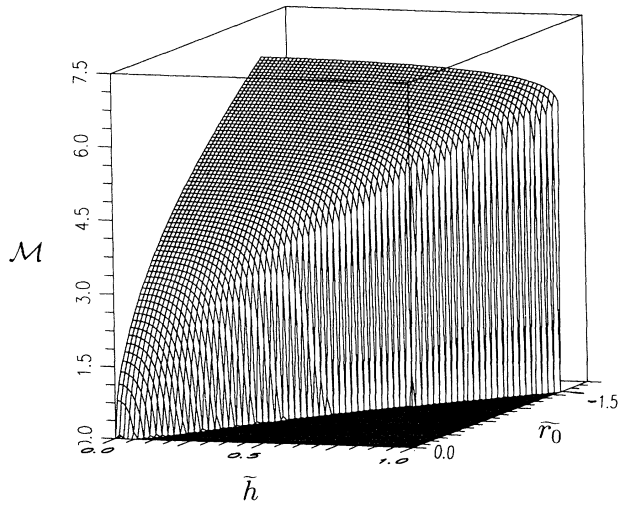

FIG. 3. $\mathcal{M}(r_0,h)$ as calculated with a Runge-Kutta scheme. Of note is the sharp slope of $\mathcal{M}(h)$ for lower temperatures. At $\tilde{h}=0$ the equilibrium magnetization is recovered.

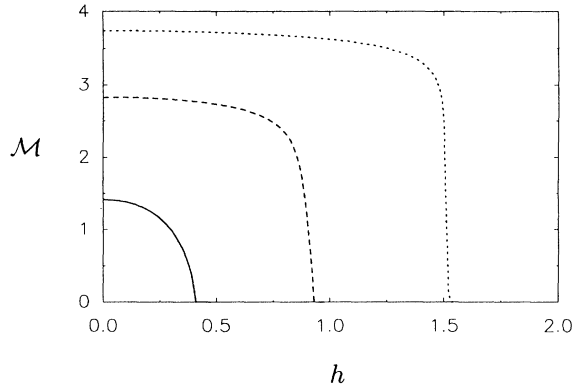

FIG. 4. $\mathcal{M}(h)$ at $(1/2\pi)(\Omega/\Gamma_0)=0.1$ at $-r_0=0.1$, 0.4, and 0.7, corresponding to the solid, dashed, and dotted lines, respectively. The general similarities of shape between these and the curves of Fig. 3 can be understood from a scaling relation $\mathcal{M}$ obeys [see Eq. (4.14)].

h axes, a plot of $\mathcal{M}(h)$ at a given temperature and frequency is similar in shape to $\mathcal{M}(h)$ at a higher temperature but lower frequency.

There are two limits where one would expect to recover equilibrium results from the phase diagram $\mathcal{M}(r_0,h)$.

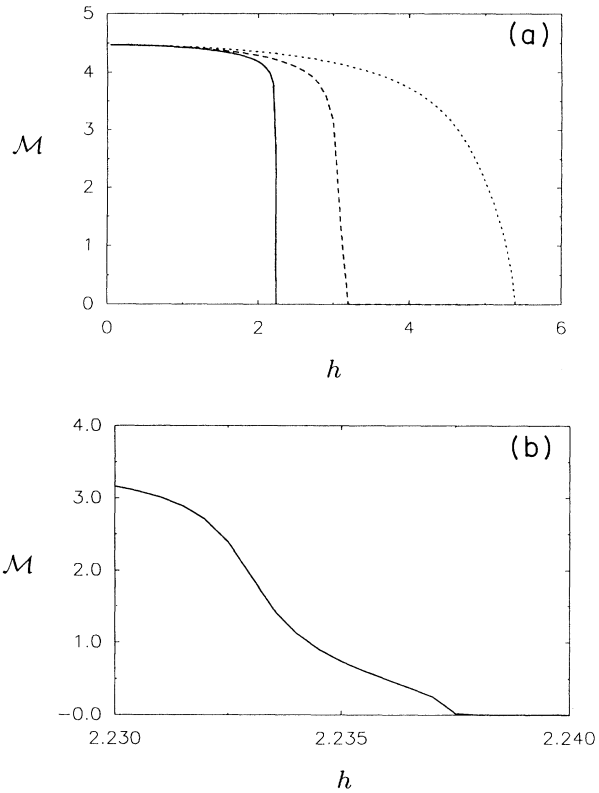

FIG. 5. (a) $\mathcal{M}(h)$ for $r_0=-1.0$ and $(1/2\pi)(\Omega/\Gamma_0)=0.1$, 0.2, and 0.3, corresponding to the solid, dashed, and dotted lines, respectively. In the first case it *appears* as though $\mathcal{M}$ changes discontinuously, although it actually does not. (b) An expanded view of the first case of (a), showing that $\mathcal{M}$ increases continuously. The range of h for which this initial slow increase takes place shrinks with lower temperature.

As $h \to 0^+$, $\mathcal{M}$ approaches the static magnetization $\pm(-r_0/u_0)^{1/2}$. This limiting case occurs smoothly because, as already mentioned, when h is small $M(t)$ consists of a static part [which is approximately $\pm(-r_0/u_0)^{1/2}$] and a small oscillating piece; as $h \to 0^+$ the oscillating piece vanishes. Also, in the limit $\Omega \to 0$ one might (naively) expect that the effect of a static external field on the Ising model would be recovered. While this is true if one is considering $M(t)$ with equilibrium initial conditions, this is not the case if one is working with $\mathcal{M}(r_0,h)$. The reason is that (1) it is necessary to take $\Omega t \to 0$, and not just $\Omega \to 0$ in order to recover the initial condition [16] and (2) the order parameter $\mathcal{M}$ always involves Ωt up to 2π, since it is defined as $M(t)$ integrated over one period. Hence, it is not possible to recover the desired static result.

V. APPROXIMATE SOLUTIONS

Here a simple linear response and a Fourier solution is used to approximately calculate $M(t)$ in the Z phase. The linear-response solution is

$$M_0(t) = \Gamma_0 \int_{-\infty}^{t} e^{-\omega_0(t-t')} h(t')dt' = \frac{h\Gamma_0}{(\Omega^2+\omega_0^2)^{1/2}} \cos(\Omega t - \theta) , \quad (5.1)$$

where $\omega_0 = \Gamma_0 r_0$ and $\tan\theta = \Omega/\omega_0$. The Fourier expansion involves the series $M_F(t) = \sum_n m_n \cos(n\Omega t + \varphi_n)$. To lowest order, m_1 is defined through

$$h^2 = m_1^2(\tfrac{3}{4}u_0 m_1^2 + r_0)^2 + (\Omega/\Gamma_0)^2 m_1^2 . \quad (5.2)$$

These approximate solutions are substituted into $r_0 + 3u_0 M^2 = 0$ ($M = M_0, M_{NZ}$) to get equations for the phase boundary curves (see Fig. 6). Both give the correct qualitative behavior, but have quantitative differences. In both approximate solutions $-r_{0c} \approx A u_0 h^2 (\Gamma_0/\Omega)^2$ as $r_0 \to 0$, and $-r_{0c} \approx B(u_0 h^2)^{1/3}$ as $r_0 \to -\infty$. For the linear-response solution $A = \frac{3}{2}$ and $B = (\frac{3}{2})^{1/3}$, while for the Fourier solution $A = \frac{3}{2}$ and $B = (\frac{3}{2})^{1/3}$. These scaling behaviors are in agreement with that from dimensional analysis [see Eqs. (3.10) and (3.12)] and numerical simulations [see Eqs. (4.1) and (4.2)].

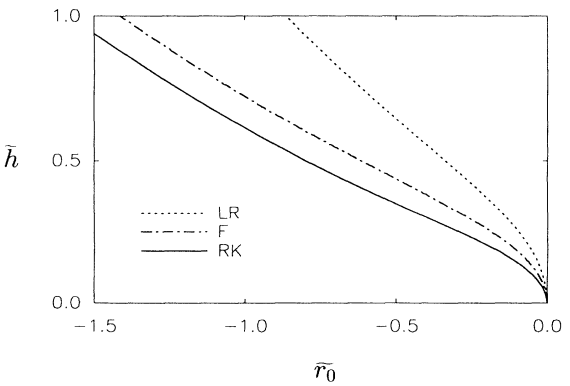

FIG. 6. Phase boundary lines as found by approximate schemes utilizing a linear-response (LR) and a Fourier (F) expansion solution for $(1/2\pi)(\Omega/\Gamma_0) = 0.5$. The Runge-Kutta (RK) solution is included for comparison.

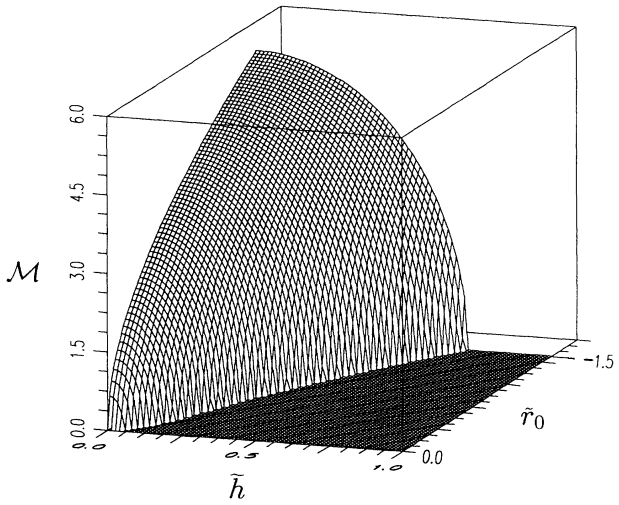

FIG. 7. Calculation of $\mathcal{M}(r_0,h)$ using a simple approximation scheme. The qualitative shape is reproduced but the sharp dropoff is not seen.

Likewise, in the NZ phase it is possible to calculate the average magnetization in an approximate way. Taking the linear-response solution as the lowest-order solution for $\tilde{m}$, it is used in conjunction with Eq. (3.2) to find $\mathcal{M}(r_0,h)$ (see Fig. 7). While there is rough qualitative agreement between this and the plot obtained using a Runge-Kutta scheme, this approximate calculation does not show the same sharp dropoff for lower temperatures.

VI. COMPARISON TO PREVIOUS WORK

For systems not too far from equilibrium it is not unreasonable to assume a Boltzmann distribution in describing probabilities of fluctuations. More specifically, this is a reasonable choice if there are a sufficient number of spin flips during one period of oscillation. This amounts to the requirement that $\omega \gg \Omega$, where ω is the effective spin-relaxation rate; indeed, this is one of the implicit assumptions of Glauber dynamics. However, this assumption cannot be expected to always hold true, for instance, near the phase boundary where $\omega \to 0$ (due to critical slowing down) and Ω remains finite. To roughly indicate where this condition ($\omega \gg \Omega$) breaks down, the equation $\omega/\Omega = \frac{1}{2}$ is plotted in Fig. 8 in the Z and NZ phases (i.e., for $\omega = \omega_Z, \omega_{NZ}$, respectively). What is seen is that Glauber dynamics is not valid within a region bounding the phase boundary, and this region shrinks for lower temperatures. The conclusion is that the use of Glauber dynamics by Tomé and de Oliveira [1] in describing the Z-NZ phase boundary cannot be justified. Thus, if there is any inconsistency between their work and that presented here, it would be expected they are in error, due to the breakdown of the condition $\omega \gg \Omega$. Indeed, in their work there was an overlap of the Z and NZ phases, implying a discontinuous transition for low enough temperatures as well as a dynamical tricritical

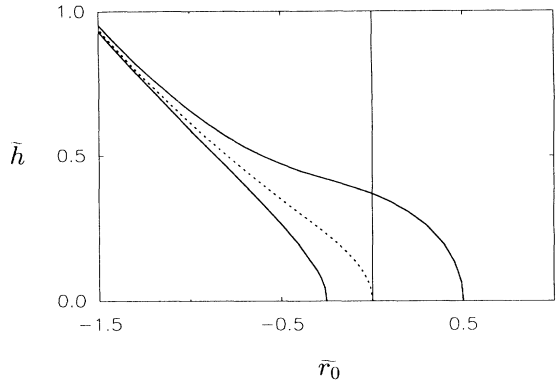

FIG. 8. The dotted line is the phase boundary curve of Fig. 2. The upper and lower solid lines are where $\omega_Z/\Omega=\frac{1}{2}$ and $\omega_{NZ}/\Omega=\frac{1}{2}$, respectively. Between them (i.e., where $\omega_{Z,NZ}/\Omega<\frac{1}{2}$) is where Glauber dynamics is invalid because the spins do not have sufficient time to relax during a time $\sim\Omega^{-1}$.

point. Such a result was not found in this treatment, and so it is not expected to be true. Finally, it is interesting to note that these two different results should nearly coincide for low temperatures, since the above-mentioned region shrinks to a great extent. In fact, as was shown in Fig. 3, $\mathcal{M}$ drops very steeply in that region. So although $\mathcal{M}(h)$ seems as though it will be discontinuous in h, it is actually continuous, as has been shown (at least for the range of parameters studied).

Finally, it is noted that the simulations that have been done to date have only been able to confirm the existence of the Z and NZ phases and the sudden change in $\mathcal{M}$ for lower temperatures. They have not been accurate enough to be able to discern a dynamical tricritical point. Also, there have been studies of the scaling behavior of the area of a hysteresis loop by Monte Carlo simulations [3,4,17] and cell dynamical simulations [18].

VII. SUMMARY

In this paper I have studied a spin system forced from equilibrium by a time-dependent external field. At least for this particular case, means were found that were useful in characterizing the different phases as well as their stability. Instead of relying on an approximate form of the nonequilibrium distribution functional (of which no general form is known), the mean-field behavior was modeled with a phenomenological equation of motion. This was used to compute the time-averaged magnetization ($\mathcal{M}$) as a function of temperature and h; a length-scale (formed from dynamic parameters only) was important in determining the overall shape of the phase diagram of $\mathcal{M}$.

As opposed to a previous work which found discontinuous and continuous transitions in $\mathcal{M}$ as a function of h, I found the transition to always be continuous. The previous method was based on Glauber dynamics (GD), which was here argued to become invalid near the phase-transition line. Because GD utilizes the equilibrium distribution, one can only expect it to possibly be a useful approximation in some limiting cases. Indeed, it was seen that GD only agreed with the work presented here in the adiabatic approximation, when the time scale of the external field is much greater than that of the magnetization.

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